

The

gErmaN dErINg

Index



Using geospatial analysis to measure relative compactness of electoral districts.

An Azavea White Paper

October 2006

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There are many factors contributing to these electoral ills, but

Introduction

If you are voting in congressional elections this fall and you live in many parts of the United States, the chances are good that your vote does not matter. In fact, you may already know this and so will decline to vote. The United States has one of the lowest voter turnout rates of any democracy in the world (54% in years of presidential elections and under 40% in mid-term elections). Further, few districts are competitive with only four Congressional incumbents losing to challengers in 2002, the fewest in history. ¹

one of them, gerrymandering — the practice of crafting district boundaries for political gain — appears to be getting worse. Recent battles in Texas, California, Georgia and New York have highlighted the increasing sophistication with which the political parties carry out the practice. In Texas, after Republican House Majority Leader Tom DeLay led a 2003 effort to gerrymander the previously approved 2002 districts, Democratic legislators fled to Oklahoma and New Mexico in an attempt to prevent a legislative quorum. The Republican gerrymander was seen as payback for the Democrats gerrymandering of the districts after the 1990 census. The plan was approved, but led to a Supreme Court challenge. In its June 2006 decision, the Supreme Court validated the Texas redistricting. The 7-to-2 decision allows redrawing of districts to occur as often as a state chooses, so long as it does not harm minorities by violating the 1965 Voting Rights Act. In New York, Republicans in the northern part of the state maintain a perpetual majority in the State Senate by incorporating large prison populations located there when determining population, but with the clear understanding that the prison inmates will not be able to vote. In Georgia, Republicans took control of the state government in 2004 and promptly re-drew the previous Democratic gerrymander in 2005. The Democrats have been accused of doing the same in Maryland in 2002.

Gerrymandering affects election outcomes in a number of

ways:

- *Reduces Electoral Competition* — gerrymandering creates larger margins of victory and enables the creation of ‘safe seats’.
- *Reduces Voter Turnout* — as the chance of affecting the outcome of an election is diminished, the number of voters is reduced and campaigns have few incentives to increase turnout.
- *Outcomes Determined in Primaries* — since many seats are decided in the party primary election, only registered party members receive a meaningful vote. This can also indirectly lead to a more partisan political dialogue - if there are more contests decided in the primaries, partisan stances on a range of issues will tend to dominate since party members are effectively the only voters.
- *Increases Incumbent Advantage* — incumbents are often both engineering the gerrymandering and are the beneficiaries of it.

So we know gerrymandering happens and we know some of its effects. Why would Azavea, a software development firm, research this topic? In 2005 Azavea began developing a software service that would enable some local Philadelphia non-profits to match their member addresses with the local council person representing the address in order to support political advocacy efforts. As we expanded the service beyond Philadelphia to more than fifty cities across the United States,

we also began looking at federal and state legislative districts and were struck by some of the tortuous shapes created by gerrymandering processes at all levels of government. We began to wonder if it would be possible to generate a top-ten list of “most gerrymandered districts.” This white paper is the outcome of that curiosity. It asks a few key questions:

1. **How do we measure it?** Can we create a gerrymandering index that will enable us to systematically calculate a score and thereby rank districts in a consistent manner?

2. **Where are worst examples?** We know we have some local council districts in Philadelphia (where Azavea is headquartered) that are pretty gerrymandered, but how does this compare to other cities?

3. **Is the problem getting worse?** Azavea develops web-based software that uses geospatial technology for crime analysis, real estate, government administration, social services and land conservation. But its recent application to subvert the electoral process demonstrates one way in which the same tools can be used to harm our society. We know people are using geospatial technology to conduct gerrymandering, but is the problem actually getting worse?

This white paper will focus on the development of a Gerrymandering Index, outline the methodology used to develop this index and discuss some of its strengths and shortcomings.

More on Gerrymandering

The term gerrymandering was coined in 1812 by political opponents of then-governor Elbridge Gerry in response to controversial redistricting carried out in Massachusetts by the Democratic-Republicans. The word is a portmanteau of Gerry’s name with the word salamander, a creature that one newly-created district was said to resemble. The term gerrymandering is now widely used to describe redistricting that is carried out for political gain, though it can be applied to any situation in which distortion of boundaries is used for some purpose.



Figure 1. 1812 political cartoon run in the Boston Weekly Messenger depicting the salamander-like district that inspired the term gerrymandering.

So how does it work? There are two primary strategies employed in a gerrymander: “packing” and “cracking.” Packing refers to the process of placing as many voters of one type into a single district in order to reduce their effect in other, adjacent districts. If one party can put a large amount of the opposition into a single district, they sacrifice that district, but make their supporters stronger in the nearby districts. The second technique, cracking, spreads the opposition amongst several districts in order to limit its effect. These techniques are obviously most effective when they are combined. In both cases, the goal is to create wasted votes for the opposition. Voters in the opposition party that are packed into one district will always be sure of winning that district (so the votes are wasted there), while they will be guaranteed to lose other seats (again, wasting their votes). The overall objective is to maximize the number of wasted votes for the opposition.

The opportunity to conduct gerrymandering arises from the constitutional requirement to re-apportion congressional representation based on the decennial census. The U.S. Constitution does not specify how the redistricting should occur, how-

ever, and each state is free to determine the methodology. All states have a 'contiguity rule' requiring that districts be contiguous land areas. Some states — Arizona, Hawaii, Idaho, Montana, New Jersey and Washington — mitigate the problem by requiring that the line-drawing be carried by out non-partisan commissions. But most states do not do this, and the reasons are obvious — gerrymandering tends to protect the seats of those in power. California Governor Arnold Schwarzenegger's Proposition 77 referendum in 2005 would have required an independent commission of retired judges in that state but was met with howls of protest by both parties and vigorous campaigning to defeat it.

While congressional districts have received the most media attention, gerrymandering can be seen in state assembly and city council districts as well. We can also observe a sort of "tax base gerrymandering" that can occur when a municipal government annexes a nearby community by running the municipal boundary along a highway or river in order to capture the higher tax base of an outlying suburb. Houston is an example of where this has occurred. And while the United States is one of the only western democracies that does not systematically limit the practice, accusations of gerrymandering have been leveled in Singapore, Canada, Germany, Chile, and Malaysia.

Cicero

Gerrymandered districts are typically identifiable by their torturous and obscure shapes. Thus one means of measuring the extent of gerrymandering in a district is to calculate its 'compactness'; the more compact its shape, the less likely it is to have been gerrymandered. Azavea has used this measurement and information on local and federal districts from our Cicero™ local elected official database system to create a Gerrymandering index for local and federal districts.

Azavea developed the Cicero™ Elected Official Web Services in 2005 as a cost effective and accurate way to match citizens, businesses and other organizations with their local elected officials. Cicero was designed to enable local governments, non-profit organizations and political organizations to empower their

citizens and members to engage with local elected officials and thereby influence the outcome of decisions. It has the ability to place voters into election districts on local, state and federal levels based on address information. It provides maps of legislative districts and provides information about elected officials, including contact information and committee assignments.

The backbone of Cicero's functionality is a geographic database for local and state legislative districts. There is no official repository of spatial data on local districts — Azavea obtained the local information for each city individually, through local government websites where possible and directly from municipal officials when necessary. Thus Cicero is now the leading sources of spatial information on local legislative districts, currently containing comprehensive data for more than 50 of the largest U.S. cities. It was this large collection of data that enabled Azavea to investigate gerrymandering on such a wide scale. The Congressional district boundaries were derived from those published for each congress by the Department of Commerce, Census Bureau, Geography Division. Azavea gathered district boundary data for the 104th Congress and the 109th Congress in order to enable comparison of district boundaries over time.

Compactness

The literature on gerrymandering cites a few different methodologies for determining a gerrymander. The most common is a measure of the ‘compactness’ of the polygon representing the district. A shape’s compactness is a measure of how spread out it is. Compactness can be measured by comparing the area enclosed by a shape to the area that would be enclosed by circle with the same perimeter. A second gerrymandering metric is the Symmetry Standard.² This measurement asks the question, ‘if the vote shares were reversed, would one party obtain the same electoral result as their opponents originally did?’ For this white paper, we wanted to work with both federal and local districts and therefore limited our analysis to the compactness metric, as it relies only on the geometry of the district polygon.

The compactness (C) of a given polygon can be calculated as 4π times the area (a) divided by the perimeter (p) squared ($C = 4\pi a/p^2$), providing a measure between 0 and 1. Using this ratio, a truly compact shape (a circle) would score a 1. There are several other potential measurements of compactness, but we chose to use this particular calculation because its inputs are simple and the others tend to provide similar results, particularly when ranking shapes against each other.³





Table 1 shows how common (and not-so-common) shapes would score using this measure of compactness. As you can see, the more spread out a shape, the lower its score, while the more tightly packed, the higher the score.

The Gerrymandering Index Version 1

We began construction of our Gerrymandering Index by calculating the compactness scores for each local legislative district and multiplying them by 100, giving a range of 0 — 100, with 0 being least compact. This calculation was performed on shapefiles of both local and congressional districts for most of the 50 largest cities in the country. Some cities, like Seattle and Detroit, do not have geographic districting (instead allowing all residents to vote for all local offices), and were thus excluded from our analysis.

Table 1. Compactness values for pictured shapes. ($C = 4\pi a/p^2$)

Shape and Compactness Score

	1
	.785
	.589
	.240
	.071

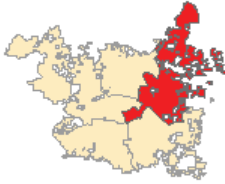

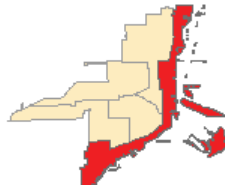

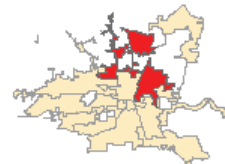
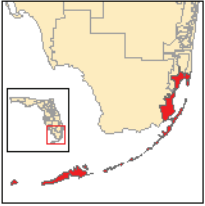
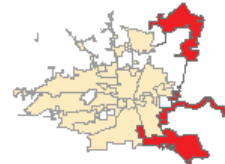
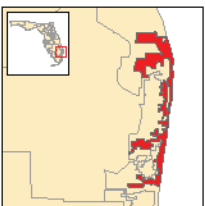
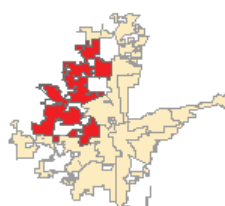
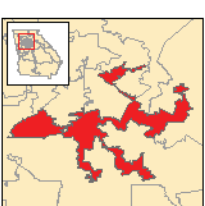
Version 1 Weaknesses

Calculating the compactness of local and federal districts revealed the following districts to be the least compact at the local and federal levels. A look at the maps of these areas quickly reveals both the strengths and weaknesses of using compactness alone as a proxy for gerrymandering. The compactness of a district can be greatly impacted by both physical features and political boundaries, and low compactness due to one of these factors would not necessarily be indicative of gerrymandering. The role of physical features can be seen quite clearly in the cases of Miami's 2nd District at the local level and Alaska at the federal level. The impact of physical geography is most obvious in coastal regions, where islands, capes and inlets add to the perimeter without corresponding increases in area, thus lowering compactness. Interestingly, this is one area where the more detailed the data (in this case, the shapefile), the more skewed the results will be. Highly generalized data, with rough estimates of coastlines, will yield much higher compactness scores than more detailed data following each twist and turn.

Raleigh, North Carolina is a good example of a city whose districts have a low score for compactness (two additional districts were in the top ten), but none of the tortuous shapes generally associated with gerrymandering. This appears to be one incidence where political boundaries at the edge of the city are creating the appearance of gerrymandering where it may not, in fact exist. Perhaps even more interesting than Raleigh is Houston, Texas, which boasts two districts among the five lowest in compactness. Unlike the case with Raleigh, Houston's districts do have convoluted shapes, with all of the odd twists and protrusions characteristic of gerrymandering. A close examination, however, reveals that even these districts follow the city boundaries, deriving their bizarre shapes from Houston's history of growth by annexation, rather than by specific manipulation of district boundaries. While politics may well have played a role in the peculiar pattern of annexation, that consideration does not fall under the category of gerrymandering.

Gerrymandering Index Version 2

Table 2. Least compact local and federal districts

Local	Federal
<p>1. Raleigh, NC – District B Compactness Value: 1.2</p> 	<p>1. California – District 23 Compactness Value: 2.5</p> 
<p>2. Miami, FL – District 2 Compactness Value: 2.5</p> 	<p>2. Alaska – District 99 Compactness Value: 2.5</p> 
<p>3. Houston, TX – District B Compactness Value: 2.5</p> 	<p>3. Florida – District 18 Compactness Value: 2.6</p> 
<p>4. Houston, TX – District E Compactness Value: 3.1</p> 	<p>4. Florida – District 22 Compactness Value: 2.7</p> 
<p>5. Ft Worth, TX – District 7 Compactness Value: 3.1</p> 	<p>5. Georgia – District 13 Compactness Value: 2.7</p> 

So, having now declared at least four of our top five local districts (based on the raw compactness ratio) to have not been gerrymandered, what does this mean for the index? Is there some way to account for the effect of municipal/state boundaries on the compactness of a district? To address this concern, we calculated the compactness values of the city (or state, in the case of federal districts) as a whole and divided the district compactness score by the city compactness score. Thus the Gerrymandering Index (GI) is now expressed as $GI = C_{\text{district}}/C_{\text{city}}$. A GI value less than 1 represents a district that is less compact than the city in which it is located, while a value greater than 1 represents a district that is more compact than its city. This measurement does run the risk of identifying moderately compact districts in highly compact cities as being more gerrymandered than districts of very low compactness that are in low or moderately compact cities. To address this concern, we used the individual district compactness to identify potentially gerrymandered areas and performed the additional analysis only on those districts. Districts were identified as being potentially gerrymandered if their individual compactness scores (C_{district}) were more than one standard deviation below the mean compactness score for all districts. (See compactness distributions and summary statistics for local and federal districts, p. 11.)

Version 2 Weaknesses

The local districts scoring the lowest on the updated Gerrymandering Index are shown in Table 3. From examining the new results, it is clear that by reflecting the municipal and state boundaries in the index score, we are seeing more locations that are likely being gerrymandered. However, at the local level, it is likely that our index still needs some work. In particular, Baltimore's 10th District is clearly heavily influenced by its border with the Chesapeake Bay. Though non-contiguity is often a sign of gerrymandering, in this case it is a result of natural boundaries. Additionally, it is likely that highly detailed data on the Chesapeake is disproportionately increasing the perimeter of the surrounding districts. Further, in New York City's 32nd District is clearly being drawn down based on the narrowness of the island. No mathematical formula is likely to adequately correct for all of this variability. As with any indicator, we suggest that the GI be used to identify areas of

Table 3. $GI = (C_{\text{district}}/C_{\text{city}})$; $C = 100 \times 4\pi a/p^2$

Most Gerrymandered Local Districts

1. Philadelphia, PA – District 7

Compactness Value: 7.62



6. Atlanta, GA – District 5

GI: 0.59 (Compactness: 12.15)



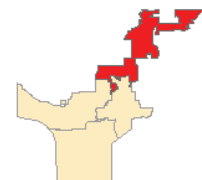
2. Nashville, TN - District 13

GI: 0.25 (Compactness: 7.62)



7. Tampa, FL – District 7

GI: 0.60 (Compactness: 8.39)



3. Philadelphia, PA – District 5

GI: 0.31 (Compactness: 12.10)



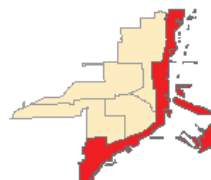
8. New York, NY – District 32

GI: 0.68 (Compactness: 9.98)



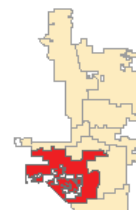
4. Miami, FL – District 2

GI: 0.42 (Compactness: 2.51)



9. Phoenix, AZ – District 7

GI: 0.69 (Compactness: 4.40)



5. Baltimore, MD - District 10

GI: 0.46 (Compactness: 4.79)



10. El Paso, TX – District 2

GI: 0.70 (Compactness: 11.90)



potential gerrymandering, but that the particulars of each case should also be used as a guide. Table 4 depicts the top 10 most gerrymandered local districts after eliminating those that remain highly influenced by municipal and natural boundaries. Table 5 depicts the most gerrymandered federal districts, none of which were eliminated based on boundary considerations.

Table 4. $GI = (C_{\text{district}}/C_{\text{city}})$; $C = 100 \times 4\pi a/p^2$

Most Gerrymandered Local Districts — Modified

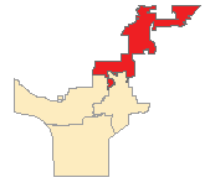
1. Philadelphia, PA – District 7

GI: 0.25 (Compactness: 7.62)



6. Tampa, FL – District 7

GI: 0.60 (Compactness: 8.39)



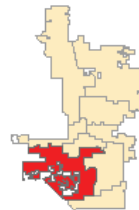
2. Nashville, TN – District 13

GI: 0.31 (Compactness: 12.10)



7. Phoenix, AZ – District 7

GI: 0.6 (Compactness: 4.40)



3. Philadelphia, PA – District 5

GI: 0.37 (Compactness: 11.54)



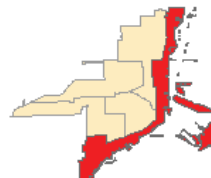
8. El Paso, TX – District 2

GI: 0.70 (Compactness: 11.90)



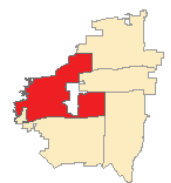
4. Miami, FL – District 2

GI: 0.42 (Compactness: 2.51)



9. Arlington, TX – District 4

GI: 0.71 (Compactness: 12.33)



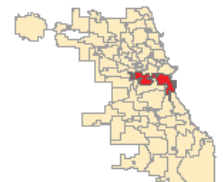
5. Atlanta, GA – District 5

GI: 0.59 (Compactness: 12.15)



10. Chicago, IL – Ward 2

GI: 0.76 (Compactness: 8.67)



10 Most Gerrymandered States

Using a similar process as that used for federal congressional districts, we determined the 10 most gerrymandered states by averaging the compactness of all districts in the state and dividing that by the compactness of the state itself. For the same reason that GI was only calculated for districts more than 1 standard deviation below the mean, GI for states was only calculated when average compactness was below the average for all states.

1. Georgia	GI = .30
2. Pennsylvania	GI = .34
3. Alabama	GI = .36
4. Ohio	GI = .44
5. Illinois	GI = .47
6. New Jersey	GI = .47
7. South Carolina	GI = .51
8. Connecticut	GI = .53
9. New Hampshire	GI = .58
10. California	GI = .59

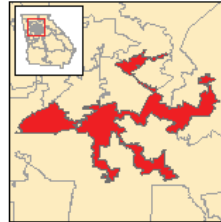
Note: Lower scores are indicative of greater gerrymandering.

Table 5. $GI = (C_{\text{district}}/C_{\text{state}})$; $C = 100 \times 4\pi a/p^2$

Most Gerrymandered Federal Districts

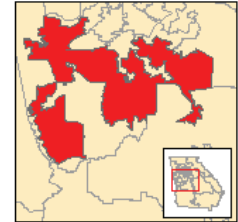
1. Georgia - District 13

GI: 0.07 (Compactness: 2.74)



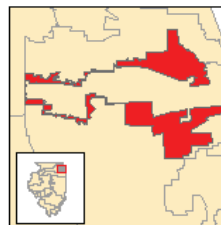
6. Georgia – District 8

GI: 0.10 (Compactness: 4.07)



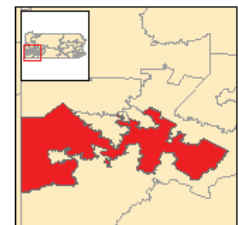
2. Illinois - District 4

GI: 0.08 (Compactness: 3.45)



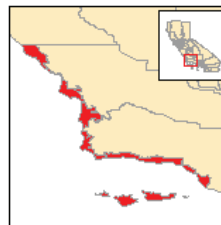
7. Pennsylvania – District 18

GI: 0.11 (Compactness: 6.04)



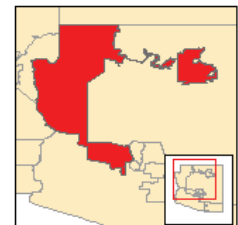
3. California - District 23

GI: 0.09 (Compactness: 2.54)



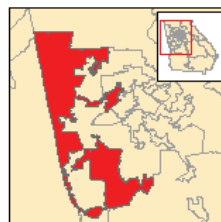
8. Arizona – District 2 *

GI: 0.13 (Compactness: 8.06)



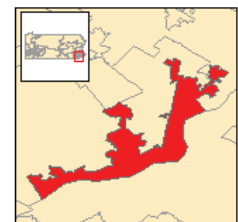
4. Georgia – District 11

GI: 0.09 (Compactness: 3.56)



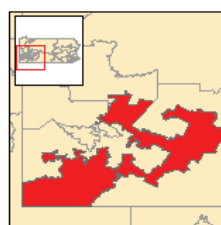
9. Pennsylvania – District 1

GI: 0.13 (Compactness: 6.73)



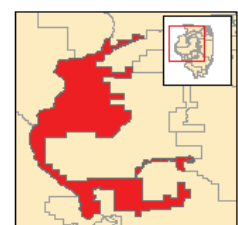
5. Pennsylvania – District 12

GI: 0.10 (Compactness: 5.00)



10. Illinois – District 17

GI: 0.13 (Compactness: 5.61)

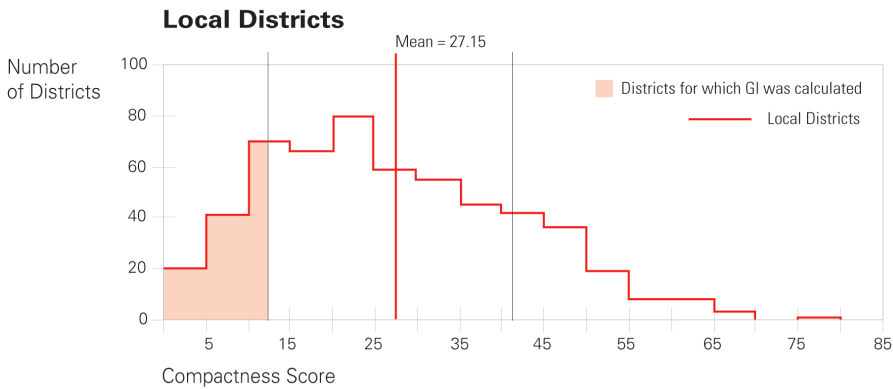


* **Note:** Arizona has used an independent redistricting commission. This shape is designed to accommodate concerns of the local Hopi tribe.

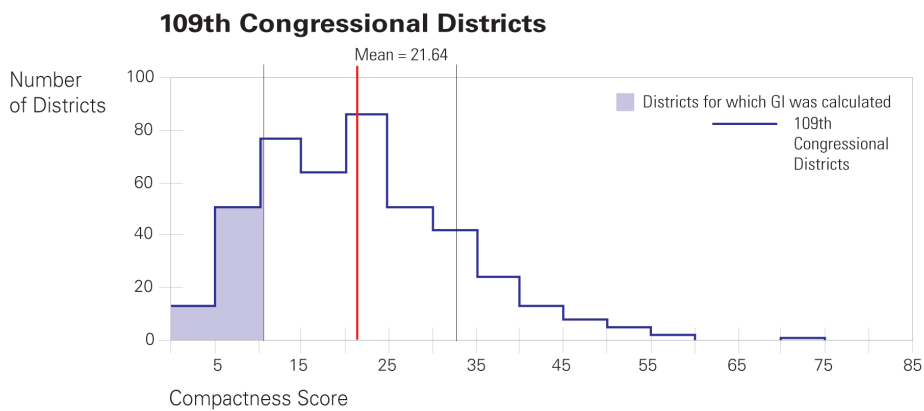
Summary Statistics for Local and Federal District Compactness

These histograms represent the distribution of compactness scores for local and federal electoral districts. Compactness scores can range from 0 to 100 with higher scores indicating more compact districts. The average compactness score is indicated in red and the blue lines represent scores that area one standard deviation above

and below the average. Only districts with compactness scores more than one standard deviation below the mean were used in the calculation of the Gerrymandering Index.



Mean	27.15
Standard Deviation	14.69
Minimum	1.98
Maximum	76.08



Mean	21.64
Standard Deviation	11.22
Minimum	2.54
Maximum	72.61

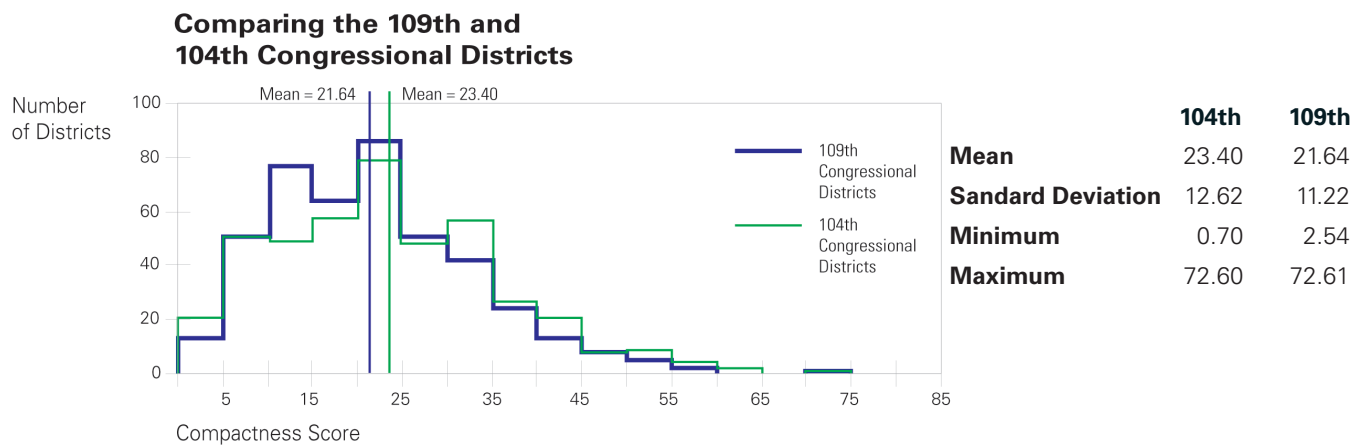
What a Difference Ten Years Make

While attempts to gerrymander political districts have existed for almost as long as geographic representation, there has been concern in recent years that the widespread availability of desktop GIS technology as well as specialized redistricting tools has encouraged a more pervasive use of gerrymandering as a technique for both of the major political parties to acquire and retain political power. When combined with detailed demographic data about households as well as detailed databases of party registration, campaign donations and poll attendance, it has become possible to predict aggregate voter outcomes with substantial precision. These tools have enabled political parties to dramatically increase the efficiency of their gerrymandering efforts.

There is no question that elections in the U.S. House of Representatives have become less competitive in recent years with fewer seats decided by margins of less than 10%. But do we see an increase in the amount of gerrymandering reflected in the geometry of the districts? In trying to answer this question, Azavea analyzed the shapes of congressional districts from the 104th Congress (1995 — 1996) with that of the 109th Congress (2005 — 2006). We analyzed differ-

ences in the compactness scores for the two sets of districts, finding that congressional districts are indeed less compact now than they were ten years ago. While it is beyond the scope of this paper to determine exact reasons for this change, the advances in geographic technology during the intervening years certainly provide ample support for any lawmaker with gerrymandering on his or her mind.

This histogram compares the distributions of compactness scores for federal districts during the 104th and 109th Congresses. The later districts are indeed less compact than the earlier, and since we know that individual state shapes have not changed during that time, the result is highly indicative of increased gerrymandering, potentially related to the improvement in geographic technologies during the intervening years. Though the difference in compactness between the two distributions below is not great, it is statistically significant (t-test, $p < 0.05$). It is up to the reader to determine the real-world significance of such a change.



Discussion

Several states in the United States have addressed gerrymandering problems by the establishment of independent redistricting commissions, usually composed of retired judges. While this is a positive step forward, independent redistricting commissions are rarely sufficient to guarantee a both competitiveness and fair representation. Reform organizations such as the FairVote have also called for the establishment of multi-seat ‘Superdistricts’ with selection occurring through proportional representation in order to improve both partisan balance, competitiveness, voter turnout and representation of racial minorities.

Due to the variety of factors that come into play in determining legislative boundaries, gerrymandering is rarely simple to identify. Truly bizarre and convoluted shapes can result from processes unrelated to partisan redistricting schemes. Physical landscape features from coastlines to mountain ranges impact decisions on where to draw district boundaries and unusual growth patterns create convoluted cities, rendering compact district design all but impossible. The gerrymandering index described in this white paper attempts to quantify the extent to which a local or federal district may be gerrymandered, based on its level of compactness and that of its city or state. Because of the combined impacts of political boundaries and physical geography, other factors may be taken into consideration when looking into a particular district, such as shape, contiguity and respect for political subdivisions. Nonetheless, compactness measures are a reliable indicator that gerrymandering is likely and point the way to districts worthy of higher scrutiny.

Additional Resources

Wikipedia

<http://en.wikipedia.org/wiki/Gerrymandering>
http://en.wikipedia.org/wiki/2003_Texas_redistricting

FairVote: The Center for Voting and Democracy

<http://www.fairvote.org/>
Redistricting Reform Watch 2005
<http://www.fairvote.org/?page=1389>
Mapping Our Future: A Public Interest Guide to Redistricting 2000
<http://www.fairvote.org/?page=285>

National Conference of State Legislatures

Redistricting Resources
<http://www.ncsl.org/programs/legman/elect/redist.htm>
State Legislative Redistricting Sites
<http://www.ncsl.org/programs/legman/elect/statesites.htm>

ACE Project: The Electoral Knowledge Network

http://www.aceproject.org/ace-en/topics/bd/bdy/bdy_us/

United States Elections Project, George Mason University

<http://elections.gmu.edu/>

Psephos: Adam Carr’s Election Archive

<http://psephos.adam-carr.net/>

Daily Sonic

<http://www.dailysonic.com/segment1039>
<http://www.dailysonic.com/gerrymander/index.php>

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1. Rob Ritchie, Center for Voting and Democracy (Fairvote.org) as quoted by BBC News, “Map Redrawing Angers Democrats,” October 8, 2004.
 2. Grofman, Bernard and King, Gary. The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering after LULAC v. Perry.
<http://gking.harvard.edu/files/jp.pdf>
 3. Gillman, Rick. Geometry and Gerrymandering.
<http://www.valpo.edu/mathcs/ResearchPapers/gerryandtables.pdf>